Discounting The Future - Part II
Valuing A Perpetual Bond

Gary Schurman, MBE, CFA

May, 2026

Wikipedia definition: A perpetual bond, also known as a "perp" or "consol bond," is a fixed-income security with no maturity date, meaning the issuer never repays the principal. It pays interest (coupons) indefinitely, offering a continuous stream of income that makes it **function more like equity than debt**.

Note: From the perspective of the bond issuer (i.e. borrower) where bond principal value increases over time, the borrower pays periodic coupon payments **to** the bond holder (negative cash flow) and receives periodic principal increases (positive cash flow) **from** the bond holder.

In this white paper we will value a perpetual bond. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We are tasked with valuing a perpetual bond. The table below presents our bond model parameters...

Description	Value
Bond face (principal) value	\$100,000
Number of annual coupon pmts	1
Annual bond principal growth rate	3.50 %
Annual bond coupon rate	5.75 %
Annual real rate of return	2.25 %
Annual inflation rate	2.75 %
Annual risk premium	1.70 %

We will define the variable B_t to be bond principal value at the end of time t and the variable g to be the annual bond principal growth rate. The equation for bond principal at the end of time t as a function of bond face value is...

$$B_t = B_0 \, (1+g)^t \tag{1}$$

The bond pays one coupon payment annually. We will define the variable C_t to be the coupon payment at the end of time t and the variable r to be the annual bond coupon rate. Using Equation (1) above and the table above, the bond coupon payment calculation is...

$$C_t = \text{Bond principal at the beginning of time } t \times \text{Coupon rate} = r \, B_{t-1} = r \, B_0 \, (1+g)^{t-1} \tag{2}$$

Bond principal increases at the annual bond principal growth rate. We will define the variable P_t to be bond principal received at the end of time t. Using Equation (1) above and the table above, the bond principal received calculation is...

$$P_t = \text{Bond principal at the beginning of time } t \times \text{Principal growth rate} = g \, B_{t-1} = g \, B_0 \, (1+g)^{t-1} \tag{3}$$

We will define the variable k to be the annual market interest rate. The market interest rate is the interest rate that the market demands to buy this bond. Using the table above, the market rate calculation is...

$$k = \text{Real rate of return} + \text{Inflation rate} + \text{Risk premium} = 0.0225 + 0.0275 + 0.0170 = 0.0670 \tag{4}$$

Building Our Model

We will define the variable V_0 to be the market value of our bond at time zero. The equation for bond value is...

$$V_0 = \text{Value of coupon payments} - \text{Value of principal received} \tag{5}$$

Using Equations (1), (2), (3) and (4) above, the equation for bond market value at time zero is...

$$V_0 = \frac{C_1 - P_1}{(1+k)^1} + \frac{C_2 - P_2}{(1+k)^2} + \frac{C_3 - P_3}{(1+k)^3} + ... + \frac{C_\infty - P_\infty}{(1+k)^\infty} \tag{6}$$

Note that we can rewrite Equation (6) above as...

$$V_0 = \sum_{t=1}^{\infty} \frac{C_t - P_t}{(1+k)^t} = \sum_{t=1}^{\infty} \frac{(r-g)\, B_0\, (1+g)^{t-1}}{(1+k)^t} = \frac{r-g}{1+g}\, B_0 \sum_{t=1}^{\infty} \left(\frac{1+g}{1+k} \right)^t \tag{7}$$

We will define the variable θ to be the discount factor. Using this definition, we can rewrite Equation (7) above as...

$$V_0 = \frac{r-g}{1+g}\, B_0 \sum_{t=1}^{\infty} \theta^t \text{ ...where... } \theta = \frac{1+g}{1+k} \tag{8}$$

Note the solution to the following geometric series... [1]

$$\sum_{t=1}^{\infty} \theta^t = \frac{\theta - \theta^{(\infty-1)}}{1-\theta} = \frac{\theta}{1-\theta} \text{ ...given that... } 0 < \theta < 1 \tag{9}$$

Using the definition of theta in Equation (8) above, the solution to Equation (9) above is...

$$\sum_{t=1}^{\infty} \theta^t = \frac{1+g}{1+k} \Bigg/ \left(1 - \frac{1+g}{1+k} \right) = \frac{1+g}{1+k} \Bigg/ \frac{k-g}{1+k} = \frac{1+g}{k-g} \tag{10}$$

Using Equation (10) above, the solution to Equation (8) above is...

$$V_0 = \frac{r-g}{1+g}\, B_0\, \frac{1+g}{k-g} = B_0\, \frac{r-g}{k-g} = (r-g)\, B_0\, \frac{1}{k-g} \tag{11}$$

Note that if $r = k$ then the value of perpetual debt at time zero is the principal value of that debt. Using Equation (11) above, this statement in mathematical terms is...

$$V_0 = (k-g)\, B_0\, \frac{1}{k-g} = B_0 \text{ ...when... } r = k \tag{12}$$

The Answer To Our Hypothetical Problem

Using Equations (4) and (8) above, the equation for the value of our discount factor is...

$$\theta = \frac{1+g}{1+k} = \frac{1+0.0350}{1+0.0670} = 0.97001 \tag{13}$$

Using Equations (11) and (13) above, the value of our bond coupon payments is...

$$\text{Value of coupon payments} = r\, B_0\, \frac{1}{k-g} = 0.0575 \times 100,000 \times \frac{1}{0.0670 - 0.0350} = 179,687.50 \tag{14}$$

Using Equations (11) and (13) above, the value of our bond principal payments is...

$$\text{Value of principal payments} = g\, B_0\, \frac{1}{k-g} = 0.03500 \times 100,000 \times \frac{1}{0.0670 - 0.0350} = 109,375.00 \tag{15}$$

Using Equations (5), (14) and (15) above, the value of our term bond is...

$$\text{Bond market price} = 179,687.50 - 109,375.00 = 70,312.50 \tag{16}$$

References

[1] Wikipedia - List of mathematical series